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Phase transition in multi-scalar-singlet extensions of the Standard Model



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ABSTRACT

We propose a generalization of the Standard Model (SM) by adding two real gauge-singlets S_1, S_2 . The field S_1 will improve the strength of the electroweak phase transition (EWPT). Imposing a Z_2 symmetry on the field S_2 makes this field a possible candidate for dark matter. Both singlets interact with other observable fields through the Higgs boson. They are allowed to interact with each other as well. We find that by introducing two different scalar fields, the model is less vulnerable to experimental constraints. In this paper, we consider the effects of a heavy scalar ($M_1 > M_H$) on the electroweak phase transition. And we present configurations that produce a strong first order EWPT.

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1. Introduction

A solution to explain the baryon asymmetry of our universe [1] is based on violation of baryon number, C and CP violation as well as a departure from thermal equilibrium. In a viable model of electroweak baryogenesis the departure from thermal equilibrium is realized via a strong first order phase transition [2]. But the SM of particle physics cannot provide a strong first order phase transition [3]. In addition the SM does not have a candidate for dark matter (DM) as well.

Moreover, from the three-loop β function for the Higgs self coupling it is found that the Higgs vacuum is no longer stable beyond the scale 10^{10} GeV. Hence we expect some new physics to appear before this scale [4].

Therefore, some new models are required to address these issues. A popular model is to couple a singlet scalar to the Higgs boson [5–16].

In Ref. [17] a scheme for classifying models of the electroweak phase transition has been presented. One may associate the formation of gravitational waves to a strong first-order phase transition [18–21].

But models with an addition of one real singlet cannot address all of the shortcomings of the SM. An interesting class of models is the multi-singlet extensions of the SM models [22–28]. In these

models there are more opportunities to satisfy the constraints imposed by experimental findings [29].

In order to study the dynamics of the electroweak phase transition EWPT one has to resort to techniques from the domain of thermal field theory [30–33]. An essential element is finite-temperature effective potential, which is a measure of the free energy density of the system. Generally a loop-level analysis, in conjunction with a vigorous Monte-Carlo scan of the parameter space is needed to unravel the structure of EWPT. However in this work we study the dynamics of the EWPT at the tree level.

The plan of this paper is as follows:

In Section 2 we propose a new model composed of two different gauge singlet scalar fields with coupling to the Higgs boson, they can have mutual interaction as well. We impose a discrete symmetry on only one of them. Hence this field will be a candidate for dark matter. The other field will provide us a strong first order phase transition. And we study the parameter space of the model. In Section 3 we discuss the finite-temperature potential and explain the origin of the strongly first-order phase transition at the tree level. In Section 4 we discuss the phenomenological implication of our model. And finally in Section 5 we present our conclusions.

2. The model

We propose a new extension of the SM by addition of two real gauge singlet scalars S_1 and S_2 . The Lagrangian of the scalar sector of our model is given by

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$$L_S = (D^\mu H)^\dagger (D_\mu H) + \frac{1}{2} \partial^\mu S_1 \partial_\mu S_1 + \frac{1}{2} \partial^\mu S_2 \partial_\mu S_2 - V(H, S_1, S_2), \quad (1)$$

where H denotes the complex Higgs doublet, $H^T = (\chi_1 + i\chi_2, \varphi + i\chi_3)/\sqrt{2}$.

As pointed out in Ref. [11], the most general (renormalizable) tree-level potential for the SM Higgs field H and the singlet S_1 depends on 8 parameters. Three more parameters are needed for the field S_2 , which has a Z_2 symmetry. And another extra parameter as we allow interaction between the gauge singlets S_1 and S_2 . Hence, our potential depends on 12 parameters and it is given by

$$V(H, S_1, S_2) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2 + \kappa_0 S_1 + 2(\kappa_1 S_1 + \kappa_2 S_1^2 + \kappa_3 S_2^2) H^\dagger H + \frac{1}{2} m_1^2 S_1^2 + \frac{\lambda_1}{4} S_1^4 + \kappa_4 S_1^3 + \frac{1}{2} m_2^2 S_2^2 + \frac{\lambda_2}{4} S_2^4 + \kappa_5 S_1 S_2^2. \quad (2)$$

However, one can always remove the linear term $\kappa_0 S_1$ by a redefinition of the field S_1 by a constant shift [11]. Therefore, we do not include this linear term in this work.

After symmetry breaking the fields χ_1, χ_2, χ_3 becomes the longitudinal degrees of freedom of the weak gauge bosons. The field S_1 is to improve the strength of the phase transition. And the singlet S_2 has a Z_2 symmetry. Hence this singlet is a dark matter candidate. Therefore at zero temperature the vacuum associated with this field must also be Z_2 symmetric. Hence vacuum expectation value (vev) of this field must vanish.

At $T = 0$ we can parameterize the scalar fields of our model by

$$H = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}, \quad S_1 = s_1 + x \quad \text{and} \quad S_2 = s_2, \quad (3)$$

with $v \simeq 246$ (GeV) and the parameter x is the vev of the gauge singlet S_1 .

By expanding around the minimum we obtain the squared mass matrix

$$M^2 = \begin{pmatrix} 2\lambda v^2 & 2\kappa_1 v + 4\kappa_2 s_1 v & 0 \\ 2\kappa_1 v + 4\kappa_2 s_1 v & m_1^2 + 3\lambda_1 s_1^2 + 2\kappa_2 v^2 + 6\kappa_4 s_1 & 0 \\ 0 & 0 & m_2^2 + 2\kappa_3 v^2 + 2\kappa_5 s_1 \end{pmatrix}. \quad (4)$$

At zero temperature the effective potential of the scalar sector of our model is

$$V(T=0) = -DT_0^2 h^2 + \frac{\lambda}{4} h^4 + \frac{1}{2} m_1^2 s_1^2 + \frac{\lambda_1}{4} s_1^4 + \kappa_1 h^2 s_1 + \kappa_2 h^2 s_1^2 + \kappa_4 s_1^3 + \frac{1}{2} m_2^2 s_2^2 + \kappa_3 h^2 s_2^2 + \kappa_5 s_1 s_2^2 + \frac{\lambda_2}{4} s_2^4. \quad (5)$$

It is convenient to express the parameters of the potential in terms of the physical masses M_1 and M_2 of the singlet scalars, the mixing angle θ of the singlet S_1 and the Higgs field. From the mass matrix squared we get

$$m_1^2 = M_1^2 \cos^2(\theta) + M_H^2 \sin^2(\theta) - 3\lambda_1 s_1^2 - 2\kappa_2 v^2 - 6\kappa_4 s_1, \\ M_2^2 = m_2^2 + 2\kappa_3 v^2 + 2\kappa_5 s_1, \quad \lambda = \frac{M_H^2 \cos^2(\theta) + M_1^2 \sin^2(\theta)}{2v^2}, \\ 4(\kappa_1 + 2\kappa_2 s_1)v = (M_H^2 - M_1^2) \sin(2\theta) \quad (6)$$

and by minimizing the scalar potential we obtain

$$DT_0^2 = \frac{\lambda v^2 + 2\kappa_1 s_1 + 2\kappa_2 s_1^2}{2} \\ 0 = m_1^2 s_1 + \lambda_1 s_1^3 + \kappa_1 v^2 + 2\kappa_2 v^2 s_1 + 3\kappa_4 s_1^2. \quad (7)$$

Hence

$$\lambda = \lambda_{SM} - \frac{(M_H^2 - M_1^2) \sin^2(\theta)}{2v^2}. \quad (8)$$

2.1. The parameter space of the model

From previous section we know that the parameter space of the model consists of $(\theta, M_1^2, M_2^2, \kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ and $\lambda_2)$.

In order to have a stable potential we must have [11]

$$\lambda > 0, \quad \lambda_1 > 0, \quad \lambda_2 > 0, \\ \kappa_2 > -\frac{\sqrt{\lambda\lambda_1}}{2} \quad \text{and} \quad \kappa_3 > -\frac{\sqrt{\lambda\lambda_2}}{2}. \quad (9)$$

But, if all the eigenvalues of the mass matrix eq. (4) are positive then the corresponding extremum point is a local minimum. Thus, $m_1^2 + 3\lambda_1 s_1^2 + 2\kappa_2 v^2 + 6\kappa_4 s_1 > 0$.

Similarly $m_2^2 + 2\kappa_3 v^2 + 2\kappa_5 s_1 > 0$. Hence from eq. (9) we have

$$m_1^2 + 3\lambda_1 s_1^2 + 6\kappa_4 s_1 > -v^2 \sqrt{\lambda\lambda_1} \quad \text{and} \\ m_2^2 + 2\kappa_5 s_1 > -v^2 \sqrt{\lambda\lambda_2}. \quad (10)$$

From eqs. (9), (10) and in terms of the parameters of our model we obtain

$$-\frac{\sqrt{\lambda\lambda_1}}{2} < \kappa_2 < \frac{M_1^2}{2v^2} + \frac{\sqrt{\lambda\lambda_1}}{2} \quad \text{and} \\ -\frac{\sqrt{\lambda\lambda_2}}{2} < \kappa_3 < \frac{M_2^2}{2v^2} + \frac{\sqrt{\lambda\lambda_2}}{2}. \quad (11)$$

With $\lambda_{SM} = 0.131$, $M_H = 126$ (GeV) and $v = 246$ (GeV) the ranges of allowed values of the Higgs boson quartic coupling from eq. (8) are

$$\lambda < 0.131 \quad \text{if} \quad M_1 < 126 \text{ (GeV)}, \\ \lambda = 0.131 \quad \text{if} \quad M_1 = 126 \text{ (GeV)}, \\ \lambda > 0.131 \quad \text{if} \quad M_1 > 126 \text{ (GeV)}. \quad (12)$$

In order to take the effect of mixing angle we notice that, for a light singlet ($M_1 \ll M_H$) and the mixing angle $\cos(\theta) = 0.95$. From eq. (8) we obtain $\lambda \simeq 0.12$ and when $\cos(\theta) = 0.99$ $\lambda \simeq 0.10$. Here the Higgs boson self coupling is suppressed.

For a heavy singlet ($M_1 = 250$ GeV) and the mixing angle $\cos(\theta) = 0.95$ we obtain $\lambda \simeq 0.25$ and when $\cos(\theta) = 0.99$ the value of Higgs boson self coupling $\lambda \simeq 0.19$. Hence the Higgs boson self coupling is enhanced in this region.

Therefore, our model predicts variations of the Higgs boson quartic coupling from that of the SM to be tested by precision measurements.

The triviality bound of the two singlets model is addressed in Ref. [34] and quartic coupling remain positive up to energy scalar of about 10 TeV.

In addition to the above theoretical bounds, there are constraints from experiments. Here we present ranges for the parameters of our model

$$\begin{aligned}
& -250 \text{ GeV} < \kappa_1 < 250 \text{ GeV}, \quad -333 \text{ GeV} < \kappa_4 < 333 \text{ GeV} \\
& -0.25 < \kappa_2 < 0.25, \quad 0.0001 < \kappa_3 < 0.0025, \\
& 5 \text{ GeV} < M_1 < 650 \text{ GeV}, \\
& 0.95 < \cos(\theta) < 1, \quad 0 < \lambda < 0.3, \quad 0 < \lambda_1 < 4, \quad 0 < \lambda_2 < 4.
\end{aligned} \tag{13}$$

3. A strongly first order phase transition and experimental constraints

At high temperature the effective potential is

$$\begin{aligned}
V(T) &= D(T^2 - T_0^2)h^2 - ETh^3 + \frac{\lambda_T}{4}h^4 + \frac{1}{2}m_1^2s_1^2 + \frac{\lambda_1}{4}s_1^4 + \kappa_1h^2s_1 \\
&+ \kappa_2h^2s_1^2 + \kappa_4s_1^3 + \frac{1}{2}m_2^2s_2^2 + \frac{\lambda_2}{4}s_2^4 + \kappa_3h^2s_2^2 + \kappa_5s_1s_2^2 \\
&+ [(8\kappa_1 + 6\kappa_4 + 8\kappa_5)s_1 + (8\kappa_2 + 3\lambda_1)s_1^2 \\
&+ (8\kappa_3 + 3\lambda_2)s_2^2] \frac{T^2}{24}.
\end{aligned} \tag{14}$$

The parameters of eq. (14) are given by

$$\begin{aligned}
D &= \frac{1}{24v^2}(6m_W^2 + 3m_Z^2 + 6m_t^2 + 6\lambda v^2 + 2(\kappa_2 + \kappa_3)v^2) \\
E &= \frac{1}{8\pi v^3}(4m_W^3 + 2m_Z^3) \\
\lambda_T &= \lambda - \frac{1}{16\pi^2 v^4}(6m_W^4 \ln \frac{m_W^2}{a_B T^2} + 3m_Z^4 \ln \frac{m_Z^2}{a_B T^2} - 12m_t^4 \ln \frac{m_t^2}{a_F T^2}) \\
\ln a_B &= 3.91, \quad \ln a_F = 1.14.
\end{aligned} \tag{15}$$

Let us consider the shape of the potential. As the universe cools down at temperature above a critical temperature T_c the potential has an absolute minima. At T_c we have two degenerate minima. The symmetric vacuum is denoted by $(0, s_{1T}, 0)$ and the true vacuum is designated by $(h_c, s_{1c}, 0)$, where we assumed that the field S_2 develops a v_{ev} at a temperature above T_c [13]. The critical temperature is the on-set of the electroweak symmetry breaking, and a first order EWPT occurs from the symmetric vacuum to the true vacuum. A transition is considered as strong if $\xi = \frac{h_c}{T_c} > 0.6 - 1.6$, but this ratio for the SM is $\frac{2E}{\lambda_{SM}} = 0.23$.

In order to have a symmetric vacua

$$\begin{aligned}
& m_1^2s_{1T} + \lambda_1s_{1T}^3 + 3\kappa_4s_{1T}^2 \\
& + \frac{1}{12}[(3\lambda_1 + 8\kappa_2)s_{1T} + (4\kappa_1 + 3\kappa_4 + 4\kappa_5)]T_c^2 = 0.
\end{aligned} \tag{16}$$

The conditions for the existence of the broken vacua are

$$2D(T_c^2 - T_0^2) - 3ET_ch_c + \lambda_T h_c^2 + 2\kappa_1s_{1c} + 2\kappa_4s_{1c}^2 = 0, \tag{17}$$

and

$$\begin{aligned}
& m_1^2s_{1c} + \lambda_1s_{1c}^3 + 3\kappa_4s_{1c}^2 + \kappa_1h_c^2 + 2\kappa_2s_{1c}h_c^2 \\
& + \frac{1}{12}[(3\lambda_1 + 8\kappa_2)s_{1c} + (4\kappa_1 + 3\kappa_4 + 4\kappa_5)]T_c^2 = 0.
\end{aligned} \tag{18}$$

Finally, to have a pair of degenerate vacua the following expression must hold

Table 1

The values of the critical temperature and the strength of a strong first order EWPT for several configurations. The values of κ_3 and $\cos(\theta)$ are within the current experimental bounds.

M_1 (GeV)	κ_1	κ_2	κ_4	T_c (GeV)	β	ξ
150	-14.30	-0.0510	-0.107	174.0	8.65	1.17
250	-14.30	0.0625	-0.089	177.9	5.67	1.12
350	-10.70	0.2000	0.080	183.9	3.14	1.08
450	-7.20	0.2490	0.212	190.8	1.71	1.04
550	-5.23	0.2499	0.237	196.0	1.30	1.02

$$\begin{aligned}
& \frac{1}{2}m_1^2(s_{1T}^2 - s_{1c}^2) + \frac{\lambda_1}{4}(s_{1T}^4 - s_{1c}^4) + \kappa_4(s_{1T}^3 - s_{1c}^3) \\
& + \frac{1}{24}[(3\lambda_1 + 8\kappa_2)(s_{1T}^2 - s_{1c}^2) \\
& + (8\kappa_1 + 6\kappa_4 + 8\kappa_5)(s_{1T} - s_{1c})]T_c^2 \\
& = D(T_c^2 - T_0^2)h_c^2 - ET_ch_c^3 + \frac{\lambda_T}{4}h_c^4 + \kappa_1s_{1c}h_c^2 + \kappa_2s_{1c}^2h_c^2.
\end{aligned} \tag{19}$$

By solving eqs. (16)–(19) one can determine the variables s_{1T} , s_{1c} , T_c and h_c .

The model has a rich parameter space and it is possible to generate a strong first order EWPT with a critical temperature varying from twenty GeV up to few hundred GeV.

4. Phenomenological implications

In order to investigate the physical implications of the model, we consider two different cases.

4.1. Gauge singlets without mutual interactions

In this case $\kappa_5 = 0$.

Strong first order EWPT

In Table 1 we present several configurations. The values of the parameters κ_3 and $\cos(\theta)$ for this table are:

$$\kappa_3 = 0.001, \quad \cos(\theta) = 0.954. \tag{20}$$

Since our numerical results are obtained by using the high temperature-expanded potential, we explore models which yields critical temperature which are higher than the mass of the top quark. Hence we expect that high temperature approximation to be quantitatively reasonable.

The value of the parameter κ_3 is chosen from Refs. [15,35], in order to incorporate the most recent experimental and theoretical constraints. Due to the small value of κ_3 , the dark sector has little influence on the dynamics which leads to a strong EWPT. We also selected the value of the parameter $\cos(\theta)$ from Ref. [36].

The parameter $\beta = \frac{|V_{min}(T=0)|}{|V_{min}(T=T_c)|}$. The results of Table 1 shows that for lower values of M_1 , the electroweak broken vacuum at zero temperature lies much deeper than the electroweak broken vacuum at the critical temperature.

In the last configuration of Table 1, $T_c = 196$. Within the range of validity of high temperature expansion, we compute the temperature evolution of the v_{ev} s of the doublet and the singlet field up to T_c . We also evaluate the temperature evolution of the electroweak broken vacuum. The results are presented in Table 2. The parameter $\beta_1 = \frac{|V_{min}(T=0)|}{|V_{min}(T=T_c)|}$ and for this configuration $V_{min}(T=0) = -5.568654133 \times 10^9$ (GeV).

Dark matter considerations

As far as the issue of dark matter is concerned we notice that, the bounds of the parameter κ_3 expressed in eq. (11) are from

Table 2

Temperature evolution of the vevs of the doublet (h) and the singlet field (s_1) up to T_c , for the case of $M_1 = 550$ (GeV). The parameter β_1 is a measure of the location of the electroweak broken vacuum at temperature T .

T (GeV)	s_1	h	β_1
174.0	308.34	242.10	1.22
179.5	309.68	233.09	1.24
185.0	311.14	223.08	1.26
190.5	312.72	211.95	1.28
196.0	314.43	199.46	1.30

the stability of the potential and from the positivity of the mass squared matrix.

At present a region of interest for the mass of dark matter is the region 56.8 (GeV) $< M_2 < 63$ (GeV) [35]. We see that, a severe bound of this parameter in this region can be obtained from the decay width of $H \rightarrow S_2 S_2$. In Ref. [13] it is found that

$$\kappa_3 \leq 0.013 \left(\frac{\text{GeV}}{0.5M_H - M_2} \right)^{\frac{1}{4}}. \quad (21)$$

With $M_2 = 60$ (GeV) we get $\kappa_3 \leq 0.0096$.

Yet a more restrictive bound on this parameter (see Ref. [35]) based on the relic density of dark matter, which comes from experiments is $0.0000625 < \kappa_3 < 0.00125$.

While in Refs. [13,15] these results, which prevent the occurrence of a strong first order (EWPT) is catastrophic, in our model the small values of κ_3 are acceptable. In fact from eq. (15) we see that smaller values of κ_3 are preferred as in this case the scalar S_2 will have a minor role in the occurrence of a strong first order EWPT.

Moreover, we see that in this region the variation of the parameter λ due to singlet scalar S_1 does not have influence on the bounds for κ_3 .

Hence as far as dark matter is concerned, we find that the situation is very similar to that of Ref. [15].

4.2. Gauge singlets with mutual interactions

In this case the gauge singlets are allowed to interact with each other directly. By considering the annihilation of DM into SM particles one must include the effect of a s channel reaction mediated by the gauge singlet S_1 , which has an amplitude proportional to

$$\frac{\kappa_1 \kappa_5}{s - M_1^2 + iM_1 \Gamma_{S_1}}.$$

The parameter κ_5 must be chosen in a way that the extra s channel reaction does not alter the dark matter cross section significantly, hence we defer an investigation of this matter to a future work.

5. Conclusions

There have been some attempts to address the issues of dark matter and occurrence of a strong first order electroweak phase

transition in a single unified model [13]. However, we find that it is difficult to solve both problems with a single scalar. Hence, in this paper we have presented a new two singlet scalar models.

In our model, when the dark matter field S_2 is only coupled to the Higgs boson, we find that the phenomenology of the dark sector of the model is similar to that of a singlet scalar dark matter.

We have explored the characteristics of a strong first order EWPT in the mass range $150 \text{ GeV} < M_1 < 550 \text{ GeV}$.

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